ASTRON 329/429, Fall 2015 – Midterm exam

ASTRON 329 students must complete problems 1-4. ASTRON 429 students must complete all problems. No extra credit will be given so ASTRON 329 students should not attempt problem 5. Calculators are allowed but notes are not. The back of the exam contains some constants and equations that may be useful.

Note: Never assume that the universes considered in the problems have the same cosmological parameters as our Universe, unless it is explicitly stated to assume so.

1. Consider a flat universe containing both matter and radiation, but whose energy density is *dominated by radiation*, i.e. $\Omega_{\rm rad} \gg \Omega_{\rm m}$. How does the *matter* density $\rho_{\rm m}$ depend on time (2 points)?

2. Consider a flat universe with Hubble constant $H_0 = 100$ km/s, matter density $\Omega_{\rm m} = 0.1$, and CMB temperature $T_{\rm CMB} = 5$ K. What is the redshift $z_{\rm eq}$ of matter-radiation equality (such that $\rho_{\rm m}(z_{\rm eq}) = \rho_{\rm rad}(z_{\rm eq})$ (2 points)?

3. Consider a flat universe, similar to ours, in which $\Omega_{\rm m} + \Omega_\Lambda = 1$ with $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$ at the present time.

a) In such a universe, $a(t) \to \infty$ as $t \to \infty$, a fact that you can assume. What is the Hubble parameter $H(t)$ as $t \to \infty$ (1 point)?

b) Consider the Friedmann equation in this universe as $t \to \infty$ and obtain an explicit solution for $a(t)$ valid under the approximations applicable in this limit (1 point).

c) For a flat matter-dominated universe, we saw in class that the expansion continues forever ($\dot{a}(t) > 0$ for all t) but ever more slowly ($\dot{a}(t) \to 0$ as $t \to 0$). Does this result also apply to a $\Lambda-$dominated universe like ours (1 point)?

4. Consider a flat universe containing only pressureless matter and a cosmological constant ($\Omega_{\rm m} + \Omega_\Lambda = 1$) with present day Hubble constant $H_0 = 70$ km/s/Mpc. Suppose that the last scattering surface of cosmic microwave background photons is located at $z_{\rm LSS} = 1,000$ in this universe.

a) Focus first on the matter-dominated limit, $\Omega_{\rm m} = 1$, $\Omega_\Lambda = 0$. For this limit, calculate the comoving, luminosity, and angular diameter distances to $z_{\rm LSS}$. Express your answers in Mpc (2 points).

b) Assume now that a positive cosmological constant is introduced, so that $\Omega_\Lambda > 0$ and $\Omega_{\rm m} < 1$ (keeping $\Omega_{\rm m} + \Omega_\Lambda = 1$). Suppose that the intrinsic physical size of typical cosmic microwave background (CMB) fluctuations and $z_{\rm LSS}$ are fixed. Relative to the matter-dominated limit of part a), will the CMB fluctuations appear larger or smaller on the sky (in terms of subtended angle) in the universe containing a positive cosmological constant? Justify your answer mathematically (2 points).

5. ASTRON 429 ONLY: Consider a neutrino of mass $m_\nu = 0.1$ eV. Assume that in the very early universe, neutrinos were relativistic and in thermal equilibrium with radiation. Following the annihilation of electrons and positrons as the Universe cooled, the temperature of the cosmic photon background, $T_\gamma(z_{\rm ep})$, was boosted relative to the temperature of the neutrino background, $T_\nu(z_{\rm ep})$, so that immediately following annihilation $T_\gamma(z_{\rm ep}) = (11/4)^{1/3} T_\nu(z_{\rm ep})$.

Focus on a particular neutrino in the neutrino background with total energy at that time satisfying $E_\nu(z_{\rm ep}) \approx 3/2 k_{\rm B} T_\nu(z_{\rm ep})$, where $k_{\rm B}$ is Boltzmann's constant. The temperature of the cosmic (microwave) photon background measured today is $T_\gamma(z=0) = 2.75$ K. Use these facts to calculate the redshift $z_{\rm nr}$ at which the neutrino becomes non-relativistic, defined such that $E_\nu(z_{\rm nr}) = 2 m_\nu c^2$ (i.e., kinetic energy $\approx$ rest energy; 2 points).

The following constants and equations may be useful:

$c = 2.99792458 \times 10^{10}$ cm s^{-1}
$h = 6.6260755 \times 10^{-27}$ erg s
$\hbar = 1.05457266 \times 10^{-27}$ erg s
$G = 6.67259 \times 10^{-8}$ cm^3 g^{-1} s^{-2}
$e = 4.8032068 \times 10^{-10}$ esu
$m_{\rm e} = 9.1093897 \times 10^{-28}$ g
$m_{\rm p} = 1.6726231 \times 10^{-24}$ g
$m_{\rm n} = 1.6749286 \times 10^{-24}$ g
$k_{\rm B} = 1.380658 \times 10^{-16}$ erg K^{-1}
1 eV$= 1.6021772 \times 10^{-12}$ erg
1 AU$= 1.496 \times 10^{13}$ cm
1 pc$= 3.086 \times 10^{18}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$\epsilon_{\rm rad} = \frac{\pi^2 k_{\rm B}^4 T^4}{15 \hbar^3 c^3}$